DYNAMIC RESPONSE UNDER MOVING CONCENTRATED LOADS OF UNIFORM RAYLEIGH BEAM RESTING ON PASTERNAK FOUNDATION

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ABSTRACT

The dynamic response under moving concentrated masses of uniform Rayleigh beam resting on Pasternak foundation, with simply supported boundary condition, is investigated in this work. In order to solve the governing fourth order partial differential equation, a technique based on the generalized integral transform (GIT) is used to reduce the governing equation to a sequence of second order ordinary differential equations. A modification of Struble's technique is employed for the solution of the reduced equation. Numerical results in plotted curves are presented. It is shown from the results that as the Rotatory inertia R_0 increases, the response amplitudes of the uniform Rayleigh beam decrease for both moving force and moving mass problems. Furthermore, the results show that the response amplitudes of the uniform Rayleigh beam decrease with an increase in the values of the shear modulus G_0 for fixed values of foundation modulus K_0 and Rotatory inertia R_0. Similarly, as K_0 increases, the response amplitudes decrease but the effect of G_0 is more noticeable than that of K_0. Finally, for the same natural frequency, the critical speed for the moving mass problem is smaller than that of the moving force problem. Hence, resonance is reached earlier in the moving mass problem.

Keywords: moving mass, moving force Rayleigh beam

1. INTRODUCTION

Moving loads cause solid bodies to vibrate intensively, particularly at high velocities. Thus, the study of the response of bodies subjected to moving loads has been the concern of several researchers. Among the earliest work in this area of study was the work of Willis [13] who considered the problem of elastic beam under the action of moving load. In their study, the mass of the beam was considered much smaller than the mass of the moving load. Much later the problem of simply supported finite beams lying on an elastic foundation and traversed by moving loads was investigated by Timoshenko [12]. In his analysis, he assumed that the loads were moving with constant velocities along the beam. Furthermore, Kenny [5] took up the problem of investigating the dynamic response of infinite elastic beams on elastic foundation when the beams under the influence of a dynamic load move with constant speed. He included the effects of viscous damping in the governing differential equation of motion. More recently, Oni and Awodola [9] considered the problem of a harmonic time variable concentrated force moving at uniform velocity, over a finite deep beam.

In all the aforementioned investigations, only the force effects of the moving loads are taken into consideration. The more complicated case for which the load mass and the beam mass are of comparable magnitude remained unaddressed for several years .In this case the inertia effect of the moving load is taken into consideration. This introduces singularity in the governing differential equation which makes the dynamical problem more cumbersome.

Attempt to solve this type of dynamical problem was first made by Gbadeyann and Oni [3] whose iterative method becomes divergent in some cases. This was closely followed by Fryba [2] and Odman [8]. In their works, solutions were presented in approximate form involving rather laborious perturbation techniques. Milomir et al [6] developed a method based on Fourier analysis to solve the problem of response of beams to an arbitrary number of concentrated moving masses. The method led to an approximate rapidly converging solution readily of useful importance to design analysis and calculation. This method is not connected with any previously developed technique on this subject Remarkable among the various studies after this is the work of Stanisic and Hardin [11] in 1968.They studied the two-dimensional problem of flexural vibration of plate under the actions of moving masses. For simplicity only the term that measures the effect of local acceleration in the direction of deflection was considered. A method based on integral transformation technique was used. However, this method is only suitable for the simply supported boundary conditions. Shortly after this, the corresponding one dimensional problem of the response of beams to an arbitrary number of concentrated moving masses was solved by the same group Stanisic and Hardin [11]. Only the term that measures the effect of local acceleration in the direction of the deflection was considered and the method of solution was only suitable for simple end conditions. Gbadeyan and Oni [4] developed a theory for obtaining an appropriate analytical solution to the problem of a finite Rayleigh beam (a thick beam) under the action of moving masses. The theory advances the development of an analytical versatile technique which is based on the modified generalized finite integral transform (GFIT) and the modification of the asymptotic struble's technique. A unique feature of this elegant technique is that it is capable of solving beam problems involving any of the classical conditions. Furthermore, this technique may be modified to solve one- dimensional moving mass problem having non-classical boundary conditions.

In all the aforementioned works, structures are either not resting on any foundation model or resting in the well known Winkler foundation model. The foundation model based on Winkler's approximation model is very common in literature particularly when considering the response of beam structure under the action of moving loads. It is asserted in the Winkler [14], that the pressure P(x,t) exerted by the foundation is proportional, at every point, to the deflection V(x , t) at the same point ;

i.e.

$$P(x,t) = K_0 V(x,t) \tag{1.1}$$

where K_0 is the foundation modulus.

The Winkler model has been conveyed variously by Gbadeyan, J. A. and Oni, S. T. [3] and Aiyesimi, Y. M. [1] because it claims discontinuous in the deflections of the surface of the foundation beyond the load region (i.e. at the ends of a finite beam), which is in contradiction to observation in practice.

Emphatically speaking, the characteristic feature of the well known Winkler foundation model is the discontinuous behaviors' of the surface displacements [3] continue beyond the load region .Thus, a more realistic foundation model, which admits the continuity of the surface displacement beyond the region of the load was developed by Pasternak [10] .For this model, a second foundation constant, the shear modulus G_0, enters into the formulation, and equation (1.1) turns to

$$P_G(x,t) = -\left[G_0 \nabla^2 (x,t) - K_0 V(x,t)\right] \tag{1.2}$$

where K_0 and G_0 are foundation stiffness and shear modulus respectively. This foundation model is termed Pasternak foundation.

2. GOVERNING EQUATION

Consider the flexural motion of a uniform Rayleigh beam resting on Pasternak foundation. The governing equation of motion with damping neglected of the uniform Rayleigh beam traversed by a moving concentrated loads of mass M at constant speed c, is the fourth order partial differential equation given by

$$\frac{\partial^2}{\partial x^2}\left(EI\frac{\partial^2 V(x,t)}{\partial x^2}\right) + \mu\frac{\partial^2 V(x,t)}{\partial t^2} - \mu R_O \frac{\partial^4 V(x,t)}{\partial x^2 \partial t^2} + P_G(x,t) = P(x,t) \tag{2.1}$$

where x is the spatial coordinate, t is the time, $V(x,t)$ is the transverse displacement, E is the young modulus, I is the moment of inertia, EI is the flexural rigidity of the structure, μ is the mass per unit length of the beam, R_O is the rotatory inertia correcting factor, $P(x,t)$ is the transverse concentrated load and $P_G(x,t)$ is the foundation reaction.

The boundary conditions of the structure under consideration is arbitrary and the initial condition without any loss of generality is taken as

$$V(x,0) = 0 = \frac{\partial V(x,0)}{\partial t} \tag{2.2}$$

If the inertia effect of the moving load is considered, the load $P(x,t)$ takes the form in Gbadeyan and Oni [4]

$$P(x,t) = P_f(x,t)\left[1 - \frac{1}{g}\frac{d^2 V(x,t)}{dt^2}\right] \tag{2.3}$$

where the continuous moving force $P_f(x,t)$ acting on the beam model is given by

$$P_f(x,t) = Mg\delta[x - ct] \tag{2.4}$$

g is the acceleration due to gravity and $\frac{d^2}{dt^2}$ is an acceleration operator defined as

$$\frac{d^2}{dt^2} = \frac{\partial^2}{\partial t^2} + 2c\frac{\partial^2}{\partial x \partial t^2} + c^2\frac{\partial^2}{\partial x^2} \tag{2.5}$$

Furthermore, since the load is assumed to be of mass (m) and time (t) is assumed to be limited to that interval of time within which the mass is on the beam, that is

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$$0 \le ct \le L \tag{2.6}$$

The laplacian operator ∇^2 is defined as

$$\nabla^2 = \frac{\partial^2}{\partial x^2} \tag{2.7}$$

and $\delta(x-ct)$ is the Dirac delta function defined as

$$\delta[x-ct] = \begin{cases} 0, & x \ne ct \\ \infty, & x = ct \end{cases} \tag{2.8}$$

with the properties

1. $\delta(-x) = \delta(x)$ (2.9)

2. $\int_a^b \delta[x-ct] f(x)dx = \begin{cases} 0, & ct < a < b \\ f(ct), & a < b < ct \\ 0 & a < b < ct \end{cases}$ (2.10)

In mechanics, the Dirac delta function may be thought of as a unit concentrated force acting at a point $x = 0$ Gbadeyan and Oni [4].

The uniform Rayleigh beam under consideration is assumed to be uniform, which implies, the beams parameters such as young's modulus (E), the moment of inertia (I) and the mass per unit length μ of the beam do not vary throughout the span (L) of the beam. Using equation (1.2) and (2.2) to (2.7) in equation (2.1) and after some simplification and rearrangement one obtains:

$$\frac{EI}{\mu}\frac{\partial^4 V(x,t)}{\partial x^4} + \frac{\partial^2 V(x,t)}{\partial t^2} - \frac{R_0}{\mu}\frac{\partial^2 V(x,t)}{\partial x^2} - \frac{G}{\mu}\frac{\partial^2 V(x,t)}{\partial x^2} + \frac{KV(x,t)}{\mu} - \frac{M}{\mu}\frac{\delta(x-ct)}{\partial x^2}$$

$$\left[\frac{\partial^2 V(x,t)}{\partial t^2} + 2c\frac{\partial^2 V(x,t)}{\partial x \partial t^2} + c^2\frac{\partial^2 (x,t)}{\partial x^2}\right] = \frac{mg}{\mu}\delta(x-ct) \tag{2.11}$$

3. SOLUTION PROCEDURES

Equation (2.11) is a fourth order partial differential equation with singular and variable coefficients. In this section, a general approach is developed in order to solve the initial value problem. The approach involves expressing the Dirac delta function as a Fourier cosine series and then reducing the fourth order partial differential equation (2.11) using the generalized finite integral transform (GFIT). The resulting transformed differential equation is then simplified using the modified struble's asymptotic technique.

The generalized finite integral transform is defined by

$$U(m,t) = \int_0^L V(x,t)V_m(x)dx \tag{3.1}$$

with the inverse

$$V(x,t) = \sum_{k=1}^{\infty} \frac{\mu}{U_m(x)} U(m,t)V_m(x) \tag{3.2}$$

where

$$U_m = \int_0^L \mu V_m^2(x)dx \quad (3.3)$$

and $V_m(x)$ is any function chosen such that the pertinent boundary conditions are satisfied. Thus, the m^{th} normal mode of vibration of a uniform beam

$$V_m(x) = \sin\frac{\lambda_m x}{L} + A_m \cos\frac{\lambda_m x}{L} + B_m \sinh\frac{\lambda_m x}{L} + C_m\, Cosh\frac{\lambda_m x}{L} \quad (3.4)$$

is chosen as a suitable kernel of the integral transform (3.1) where, λ_m is the mode frequency, A_m, B_m, and C_m are constants which are obtained by substituting (3.4) into the appropriate boundary conditions.

3.1 Operational simplification

By applying the generalized finite integral transform (3.1) equation (13) can be written as:

$$B_1 V(0,l,t) + B_1 T_1(t) + U_{tt}(m,t) + B_2 T_2(t) + B_3 U(m,t) + T_4(t) + T_5(t) = \frac{mg}{\mu} T_6(t) \quad (3.5)$$

where

$$B_1 = \frac{EI}{\mu}\,,\; B_2 = \frac{R_0 + G}{\mu}, B_3 = \frac{K}{\mu} \quad (3.6)$$

$$V(0,L,t) = \left[\frac{\partial^3 V(x,t)}{\partial x^3} V_m(x) + \frac{\partial^2 V(x,t)}{\partial x^2} V_m^1(x) + \frac{\partial V(x,t)}{\partial x} V_m^{11}(x) - V(x,t) V_m^{111}(x)\right]_0^L \quad (3.7)$$

$$T_1 = \int_0^L V(x,t) V_m^{iv}(x)\, dx \quad (3.8)$$

$$T_2(t) = \int_0^L \frac{\partial^2 V(x,t)}{\partial x^2} V_m(x) dx \quad (3.9)$$

$$T_3(t) = \int_0^L \frac{m}{\mu} \delta(x-ct) \frac{\partial^2 V(x,t)}{\partial t^2} V_m(x) dx \quad (3.10)$$

$$T_4(t) = \int_0^L \frac{2cm}{\mu} \delta(x-ct) \frac{\partial^2 V(x,t)}{\partial x \partial t} V_m(x) dx \quad (3.11)$$

and

$$T_6(t) = \int_0^t \delta(x-ct) V_m(x) dx \quad (3.13)$$

In order to evaluate the integral (3.8) to (3.13), use is made of the property of the Dirac delta function as an even function to express it in Fourier cosine series namely

$$\delta(x-ct)=\frac{1}{L}+\frac{2}{L}\sum_{n=1}^{\infty}\cos\frac{n\pi ct}{L}\cos\frac{n\pi x}{L} \tag{3.14}$$

Thus, in view of (3.2), using (3.14) in (3.5), after some simplification and rearrangements one obtains

$$U_{tt}(m,t)+\left(W_m^2+\frac{k}{\mu}\right)U(m,t)-\alpha_0\sum_{k=1}^{\infty}U(k,t)T_a(k,m)+\lambda_0\left[\sum_{k=1}^{\infty}U_{tt}(k,t)T_b(n,k,m)+\right.$$
$$2\sum_{k=1}^{\infty}\sum_{n=1}^{\infty}\frac{\cos n\pi ct}{L}U_{tt}(k,t)T_c(n,k,m)+2c\sum_{k=1}^{\infty}U_t(k,t)T_d(k,m)+4c\sum_{k=1}^{\infty}\sum_{n=1}^{\infty}\frac{\cos n\pi ct}{L}U_t(k,t)T_e(n,k,m)+$$
$$\left.c^2\sum_{k=1}^{\infty}U(k,t)T_a(n,k,m)+2c^2\sum_{k=1}^{\infty}\sum_{n=1}^{\infty}\frac{\cos n\pi ct}{L}U_t(k,t)T_f(n,k,m)\right]=$$
$$\frac{p}{\mu}\left[\sin\frac{\lambda_m x}{L}+A_m\cos\frac{\lambda_m x}{L}+B_m\sinh\frac{\lambda_m x}{L}+C_m\cosh\frac{\lambda_m x}{L}\right] \tag{3.15}$$

where

$$\lambda_0=\frac{1}{\mu}(R_0+G),\ \alpha_0=\frac{M}{L\mu},\ p=Mg \tag{3.16}$$

Equation (3.15) is the transformed equation governing the problem of uniform Rayleigh beam resting on Pasternak foundation. In what follows, the special cases of equation (3.15) are considered.

3.2 Closed form solution

Case (1): The moving force problem

The differential equation describing the dynamic response of uniform Rayleigh beam under the action of concentrated moving force moving at constant velocities may be obtained from equation (3.15) by setting $\alpha_0=0$. It is an approximate model, which assumes that the inertia effect of the moving mass negligible and only the force effect of the moving load is taken into consideration, thus in this case one obtains:

$$U_{tt}(m,t)+\left(W_m^2+\frac{k}{\mu}\right)U(m,t)-\lambda_0\sum_{k=1}^{\infty}U(k,t)T_a(k,m)=\frac{p}{\mu}.\left[\sin\frac{\lambda_m x}{L}+A_m\cos\frac{\lambda_m x}{L}+B_m\sinh\frac{\lambda_m x}{L}+C_m\cosh\frac{\lambda_m x}{L}\right] \tag{3.17}$$

Equation (3.17) is coupled in $U(m,t)$ *and* $U(k,t)$ and cannot be solved exact analytically, though it yields readily to numerical technique, an approximate analytical method is desirable as solutions obtained often shed light on vital information about the vibrating system.

By the use of a modification of the asymptotic method due to struble's equation (3.17) can be re-arranged to take the form

$$U_{tt}(m,t)+\left(W_{mf}^2-\lambda_0 Ta(m,m)\right)U(m,t)-\lambda_0\sum_{\substack{k=1\\k\neq m}}^{\infty}U(k,t)Ta(k,m)$$
$$=\frac{p}{\mu}\left[\sin\frac{\lambda_m ct}{L}+A_m\cos\frac{\lambda_m ct}{L}+B_m\sinh\frac{\lambda_m ct}{L}+C_m\cosh\frac{\lambda_m ct}{L}\right] \tag{3.18}$$

where

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$$W_{mf}^2 = W_m^2 + \frac{k}{\mu}, \quad \lambda_0 = \frac{1}{\mu}(R_0 + G), and \ \ T_a(m,m) = T_a(k,m)/(\mathrm{k = m}) \tag{3.19}$$

By this technique, one seeks the modified frequency corresponding to the frequency of the free system due to the presence of moving force. An equivalent free system operator defined by modified frequency then replaces equation (3.18). Thus, we set the right-hand side of (3.18) to zero and consider a parameter $\lambda < 1$ for any arbitrary ratio defined as

$$\lambda = \frac{\lambda_0}{1 + \lambda_0} \tag{3.20}$$

so that

$$\lambda_0 = \lambda + 0(\lambda^2) \tag{3.21}$$

Substituting equation (3.21) into the homogeneous part of equation (3.18) one obtains

$$U_{tt}(m,t) + \left(W_{mf}^2 - \lambda_0 T_a(m,m)\right)U(m,t) - \lambda_0 \sum_{\substack{k=1 \\ k \neq m}}^{\infty} U(k,t) T_a(k,m) = 0 \tag{3.22}$$

when λ_0 is set to zero in equation (3.22) the solution can then be written as

$$U(m,t) = C\cos(W_{mf}t - D\ ) \tag{3.23}$$

where $C \ \ and \ \ D$ are constants.

Furthermore as $\lambda < 1$ struble's technique requires that the asymptotic solution of the homogeneous part of equation (31) be of the form

$$U(m,t) = \sigma(m,t)\ \cos\left[W_{mf}t - \beta(m,t)\right] + \lambda\beta_1 + 0(\lambda^2) \tag{3.24}$$

where $\sigma(m,t)\, and\ \beta(m,t)$ are slowly varying functions of time or equivalently

$$\frac{d\sigma(m,t)}{dt} \to O(\lambda), \ \frac{d^2\sigma(m,t)}{dt^2} \to O(\lambda^2) \tag{3.25}$$

$$\frac{d\beta(m,t)}{dt} \to O(\lambda),$$
$$\frac{d^2\beta(m,t)}{dt^2} \to O(\lambda^2) \tag{3.26}$$

To obtain the modified frequency, equation (3.24) and its derivatives are substituted into equation (3.22) and neglecting terms which do not contribute to variational equations, one obtains

$$2\sigma(m,t)\beta(m,t)W_{mf}\cos\left[W_{mf}t - \beta(m,t)\right] - 2\dot{\sigma}(m,t)W_{mf}\sin\left[W_{mf}t - \beta(m,t)\right] - \sum_{k=1}^{\infty}\lambda T_a(m,m)\sigma(m,t)$$
$$\cos\left[W_{mf}t - \beta(m,t)\right] = 0 \tag{3.27}$$

Retaining terms to $O(\lambda)$ only.

The variational equations are obtained by equating the coefficient of $\sin\left[W_{mf}t - \beta(m,t)\right]$

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and $\cos\left[W_{mf}t-\beta(m,t)\right]$ on both sides of equation (3.27)

Thus

$$-2\dot{\sigma}(m,t)W_{mf}=0 \qquad (3.28)$$

$$-\lambda T_a(m,m)\ \sigma(m,t)\ +2\sigma(m,t)\dot{\beta}(m,t)W_{mf}=0 \qquad (3.29)$$

Solving equation (3.28) and (3.29) respectively gives

$$\sigma(m,t)=C\ \ and\ \ \beta(m,t)=\frac{\lambda T_a(m,m)}{2W_{mf}}t+D \qquad (3.30)$$

where $C\ and\ D$ are constants.

By substituting (3.30) into (3.34), the first approximation to the homogeneous system is

$$U(m,t)=C\cos\left[\eta_{mf}t-\beta_m\right] \qquad (3.31)$$

where

$$\eta_{mf}=W_{mf}\left[1-\frac{\lambda T_a(m,m)}{2W_{mf}^2}\right] \qquad (3.32)$$

represents the modified natural frequency due to the presence of the moving force.

Thus, to solve the non-homogeneous equation (3.18), the differential operator which acts on $U(m,t)\ and\ U(k,t)$ is replaced by the equivalent free system operator defined by the homogeneous frequency η_{mf} , thus using (3.32) the homogeneous part of equation (3.18) can be written as:

$$\frac{d^2U(m,t)}{dt^2}+\eta_{mf}^2\ U(m,t)=0 \qquad (3.33)$$

Hence, the entire equation (31) takes the form

$$U_{tt}(m,t)+\eta_{mf}^2U(m,t)=\frac{p}{\mu}\left[\sin\frac{\lambda_m ct}{L}+A_m\cos\frac{\lambda_m ct}{L}+B_m\sinh\frac{\lambda_m ct}{L}+\ C_m\cosh\frac{\lambda_m ct}{L}\right] \qquad (3.34)$$

The general solution of (3.34) is thus given by:

$$U(m,t)=\frac{p}{\mu}\left[\frac{\eta_{mf}\sin\varepsilon_c t-\varepsilon_c\sin\eta_{mf}t}{\eta_{mf}(\gamma_{mf}^2-\varepsilon_c^2)}+\frac{A_m(\cos\varepsilon_c t-\cos\eta_{mf}t)}{\eta_{mf}^2-\varepsilon_c^2}+\frac{B_m(\eta_{mf}\sin\varepsilon_c t-\varepsilon_c\sin\eta_{mf}t)}{\eta_{mf}\left(\eta_{mf}^2-\varepsilon_c^2\right)}\right.$$
$$\left.+\frac{C_m(\eta_{mf}\sin\varepsilon_c t-\varepsilon_c\sin\eta_{mf}t)}{\eta_{mf}\left(\eta_{mf}^2-\varepsilon_c^2\right)}\right] \qquad (3.35)$$

By taking the inversion of (3.35) one obtains

$$U(m,t)=\frac{1}{\alpha_m(x)}\sum_{m=1}^{\infty}\frac{p}{\mu}\left[\frac{\eta_{mf}\sin\varepsilon_c t-\varepsilon_c\sin\eta_{mf}t}{\eta_{mf}(\gamma_{mf}^2-\varepsilon_c^2)}+\frac{A_m(\cos\varepsilon_c t-\cos\eta_{mf}t)}{\eta_{mf}^2-\varepsilon_c^2}+\frac{B_m(\eta_{mf}\sin\varepsilon_c t-\varepsilon_c\sin\eta_{mf}t)}{\eta_{mf}\left(\eta_{mf}^2-\varepsilon_c^2\right)}\right.$$
$$\left.+\frac{C_m(\eta_{mf}\sin\varepsilon_c t-\varepsilon_c\sin\eta_{mf}t)}{\eta_{mf}\left(\eta_{mf}^2-\varepsilon_c^2\right)}\right]\left[\sin\frac{\lambda_m x}{L}+A_m\cos\frac{\lambda_m x}{L}+B_m\sinh\frac{\lambda_m x}{L}+C_m\cosh\frac{\lambda_m x}{L}\right] \quad (3.36)$$

where

$$\varepsilon_c=\frac{\lambda_m c}{L} \quad and \quad \alpha_m(x)=\int_0^L V_m^2(x)dx \quad (3.37)$$

Equation (3.36) represents the transverse displacement response to forces moving with constant velocities of uniform Rayleigh beam resting on Pasternak foundation.

Case (II): The Moving Mass Problem

If the mass of the moving load is commensurable with that of the structure, the inertia effect of the moving load is not negligible. Thus, in this case $\lambda_0 \neq 0$, and the solution of the entire equation (3.15) is required.
This is termed the moving mass problem. Evidently, a closed form solution of equation (3.15) is not possible. Thus the modified struble's asymptotic method is employed to get an approximate analytical solution. We neglected the terms representing the inertia effect of the moving mass in equation (3.15) and obtained (3.17). The homogeneous part of this equation can be replaceD by a free system operator defineD by the modified frequency γ_{mf} due to the presence of moving mass.
Thus, equation (3.15) can be written in the form

$$U_{tt}(m,t)+\eta_{mf}^2U(m,t)+\lambda_0\left[\sum_{k=1}^{\infty}U_{tt}(k,t)T_b(k,m)+2\sum_{k=1}^{\infty}\sum_{n=1}^{\infty}\frac{\cos n\pi ct}{L}U_{tt}(k,t)T_c(n,k,m)+2c\sum_{k=1}^{\infty}U_t(k,t)T_d(k,m)\right.$$
$$\left.+4c\sum_{k=1}^{\infty}\sum_{n=1}^{\infty}\frac{\cos n\pi ct}{L}U_t(k,t)T_e(n,k,m)+c^2\sum_{k=1}^{\infty}U(k,t)T_a(nk,m)+2c^2\sum_{k=1}^{\infty}\sum_{n=1}^{\infty}\frac{\cos n\pi ct}{L}U_t(k,t)T_f(n,k,m)\right]=$$
$$\frac{p}{\mu}\left[\sin\frac{\lambda_m ct}{L}+A_m\cos\frac{\lambda_m ct}{L}+B_m\sinh\frac{\lambda_m ct}{L}+C_m\cosh\frac{\lambda_m ct}{L}\right] \quad (3.38)$$

As in the previous case, an exact analytical solution to the above equation is not possible. The same technique used in case (1) is employed to obtain the modified frequency due to the presence of the moving mass namely

$$\varphi_m=\frac{2\eta_{mf}^2-\alpha_1\left(\eta_{mf}^2T_b(m,m)-C^2T_a(m,m)\right)}{2\eta_{mf}} \quad (3.39)$$

where

$$\alpha_1=\frac{\alpha_0}{\alpha_0+1} \quad and \quad \alpha_0=\frac{M}{\mu L} \quad (3.40)$$

$$T_b(m,m)=T_b(k,m)\big|_{k=m},T_a(m,m)=T_a(k,m)\big|_{k=m} \quad (3.41)$$

Retaining $O(\alpha_1)$ only.

Thus, equation (3.38) takes the form

$$Z_{tt}(m,t) + \varphi_m^2 Z(m,t) = \alpha_0 Lg\left[\sin\frac{\lambda_m ct}{L} + A_m \cos\frac{\lambda_m ct}{L} + B_m \sinh\frac{\lambda_m ct}{L} + C_m \cosh\frac{\lambda_m ct}{L}\right] \tag{3.42}$$

This is analogous to equation (3.34). Thus, using similar argument as in case (1) $z(m,t)$ can be obtained which on inversion yields.

$$Z(x,t) = \frac{1}{\alpha_m(x)}\sum_{m=1}^{\infty}\alpha_0 Lg\left[\frac{\varphi_m \sin\varepsilon_c t - \varepsilon_c \sin\varphi_{mf} t}{\varphi_m(\varphi_m^2 - \varepsilon_c^2)} + \frac{A_m(\cos\varepsilon_c t - \cos\varphi_m t)}{\varphi_m^2 - \varepsilon_c^2} + \frac{B_m(\varphi_m \sinh\varepsilon_c t - \varepsilon_c \sin\varphi_m t)}{\varphi_m\left(\varphi_m^2 + \varepsilon_c^2\right)}\right.$$
$$\left. + \frac{C_m(\cosh\varepsilon_c t - \cos\varphi_m t)}{\left(\varphi_m^2 - \varepsilon_c^2\right)}\right]\left[\sin\frac{\lambda_m x}{L} + A_m \cos\frac{\lambda_m x}{L} + B_m \sinh\frac{\lambda_m x}{L} + C_m \cosh\frac{\lambda_m x}{L}\right] \tag{3.43}$$

Equation (3.43) represents the transverse displacement response to concentrated masses, moving with constant velocities of uniform Rayleigh beam resting on Pasternak foundation.

4. AN IILUSTRATIVE EXAMPLE

For illustration of results in the foregoing analysis, we provide an example on simply supported uniform Rayleigh beam. In this case, the uniform Rayleigh beam has simple supports at ends X = 0 and X = L. The displacement and the bending moment vanish. Thus

$$V(0,t) = 0 = V(L,t), \frac{\partial^2 V(0,t)}{\partial x^2} = 0 = \frac{\partial^2 V(0,t)}{\partial x^2} \tag{4.1}$$

Hence, for normal modes

$$V_m(0) = 0 = V_m(L), \frac{\partial^2 V_m(0)}{\partial x^2} = 0 = \frac{\partial^2 V_m(L)}{\partial x^2} \tag{4.2}$$

which implies

$$V_k(0) = 0 = V_k(L), \frac{\partial^2 V_k(0)}{\partial x^2} = 0 = \frac{\partial^2 V_k(L)}{\partial x^2} \tag{4.3}$$

Applying (3.2) and (3.3), one obtains

$$A_m = A_k = 0, B_m = B_k = 0, C_m = C_k = 0 \tag{4.4}$$

And the frequency equation becomes

$$\text{Sin}\lambda_m = \sin\lambda_k \tag{4.5}$$

$$\lambda_m = m\pi \text{ and } \lambda_k = k\pi \tag{4.6}$$

Thus, the moving force problem is reduced to a non-homogeneous second order ordinary differential equation.

$$U_{tt}(m,t) + \beta_{ff}^2 U(m,t) = \frac{p}{\mu}\sin\alpha_c t \tag{4.7}$$

where

$$\beta_{ff}^2 = EI\left(\frac{m\pi}{L}\right)^4 + \frac{K}{\mu} + \lambda_0 \frac{m^2\pi^2}{L^2} \text{ and } \alpha_c = \frac{m\pi c}{L} \tag{4.8}$$

Equation (4.7) when solved in conjunction with the initial conditions, one obtain an expression for $U(m,t)$ which on inversion yields

$$U(x,t) = \frac{2}{L}\sum_{m=1}^{\infty}\frac{p}{2\mu\beta_{ff}}\left[\frac{\beta_{ff}\sin\alpha_c t - \alpha_c \sin\beta_{ff} t}{(\beta_{ff}^2 - \alpha_c^2)}\right]\sin\frac{m\pi x}{L} \tag{4.9}$$

which represents the transverse displacement response to force moving with constant velocities of simply supported uniform Rayleigh beam resting on Pasternak foundation?
Following arguments similar to those in the last sections, use is made of the modified asymptotic method due to struble to obtain the modified frequency due to the presence of moving mass for the simply supported uniform Rayleigh beam given as:

$$\omega_m = \frac{\beta_{ff}^2 L^2 - (m^2 c^2 \pi^2 + \beta_{ff}^2 L^2)}{\beta_{ff} L^2} \tag{4.10}$$

Neglecting higher order terms of α_0 .thus, the simply supported moving mass problem reduces to

$$U_{tt}(m,t) + \psi_m^2 U(m,t) = \alpha_1 Lg \sin\frac{m\pi ct}{L} \tag{4.11}$$

which when solved in conjunction with the initial conditions given expression for $U(m,t)$ and on inversion gives

$$U(x,t) = \frac{2}{L}\sum_{m=1}^{\infty}\frac{\alpha_1 Lg}{2\omega_m}\left[\frac{\omega_m \sin\alpha_c t - \alpha_c \sin\omega_m t}{(\omega_m^2 - \alpha_c^2)}\right).\sin\frac{m\pi x}{L} \tag{4.12}$$

which represent the transverse displacement response to a concentrated mass moving with constant velocities of simply supported uniform Rayleigh beam resting on Pasternak foundation.

5. DISCUSSION OF CLOSED FORM SOLUTION

The response amplitude of dynamical systems such as this may grow without bond. Condition under which this happens is termed resonance conditions. It is pertinent at this junction to establish conditions under which resonance occurs. This phenomenon in structural and highway engineering is of great concern to researchers or in particular, design engineers, because, for example, it causes cracks, permanent deformation and destruction in structures. Bridges and other structures are known to have collapsed as a result of resonance occurring between the structure and some signals traversing them. Evidently a simply supported uniform Rayleigh beam resting on a Pasternak foundation and traversed by a moving force will experience resonance when

$$\beta_{ff} = \frac{k\pi c}{L} \tag{5.1}$$

while the same system traversed by a moving mass reaches the state of resonance whenever

$$\omega_{ff} = \frac{k\pi c}{L} \tag{5.2}$$

Evidently,

$$\frac{\beta_{ff}^2 L^2 - (C^2 m^2 \pi^2 + \beta_{ff}^2 L^2)\alpha_1}{\beta_{ff} L^2} = \frac{k\pi c}{L} \tag{5.3}$$

Equation (5.1) and (5.3) show that for the same natural frequency, the critical speed for the same system consisting uniform Rayleigh beam resting on Pasternak foundation and traversed by a moving mass is smaller than that traversed by a moving force.

6. NUMERICAL CALCULATION AND DISCUSSIONS OF RESULTS.

We now illustrate the analysis in this work by considering a uniform Rayleigh beam of modulus of elasticity E= 2.10924×10^{10} N/m^2, the moment of inertia I= 2.87698×10^{-3} m^4, the beam spam length L= 12.192 and the mass per unit length of the beam μ= 4501.563kg/m. The value of the foundation modulus is varied between 0n/m^3 and 4000000n/m^3, the values of Rotatory inertia R_0 is varied between 0m and 4.5m, the values of the shear modulus varied between 0N/m^3 and 9000000N/m^3 the results are as shown on the various graphs as follows:

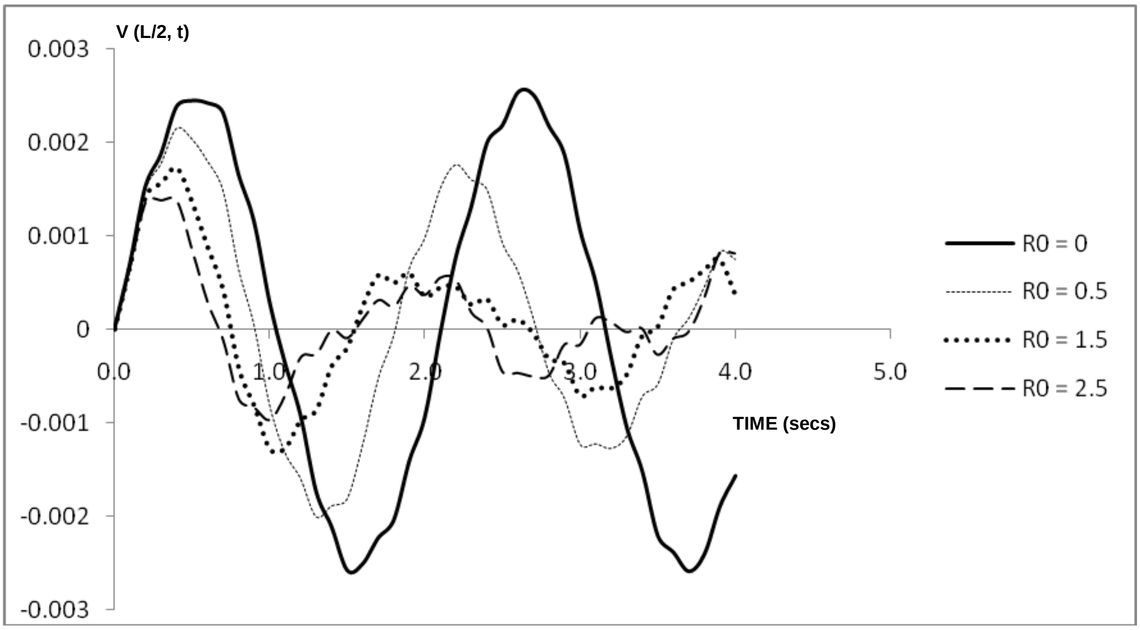

Fig 1: Transverse displacement of a simply supported Rayleigh beam under the actions of the concentrated forces travelling at constant velocity for various values of Rotatory inertia R_o and for fixed values of foundation modulus K_0 = 40000 and shear modulus G_0 = 90000.

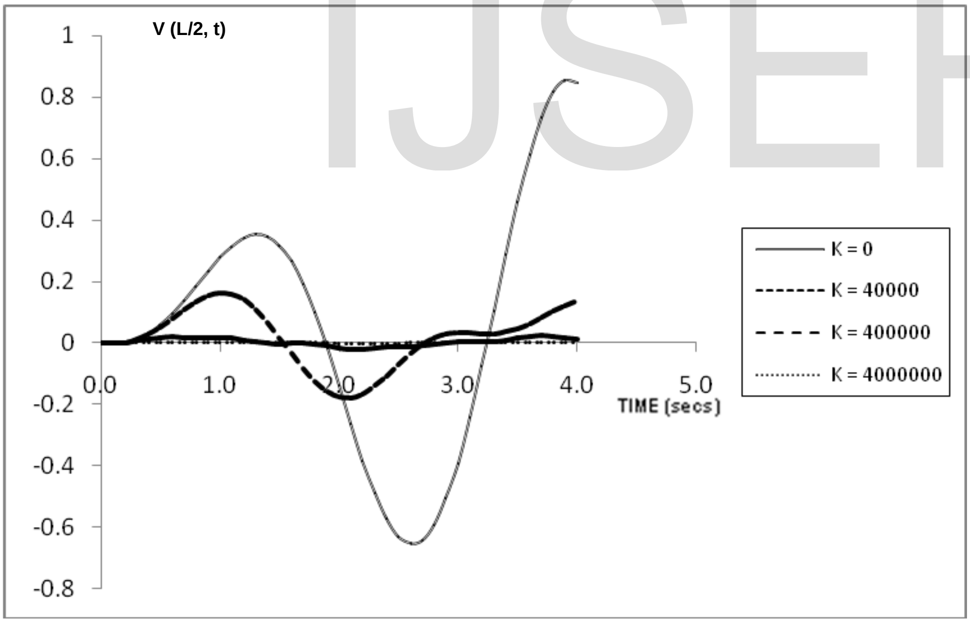

Fig 2: Deflection profile of a simply supported Rayleigh beam under the actions of concentrated forces travelling at constant velocity for various value of foundation modulus K_o and fixed values of Rotatory inertia R_o = 2.5 and shear modulus G_0 = 90000

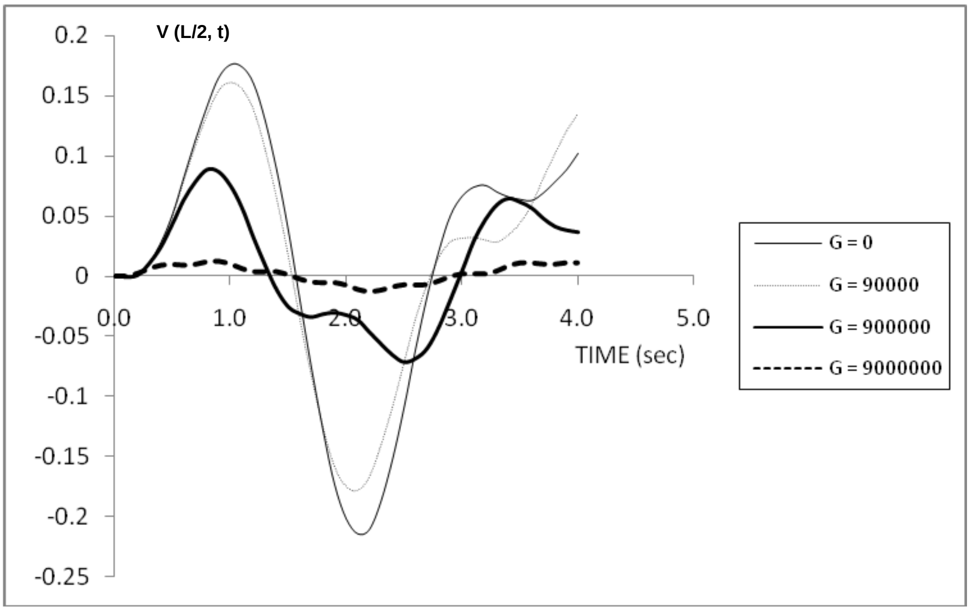

Fig3: Response amplitude of a simply supported Rayleigh beam under the actions of concentrated forces travelling at constant velocity for various values of shear modulus G_0 and for fixed values of Rotatory inertia R_o = 2.5 and foundation modulus K_0= 400000.

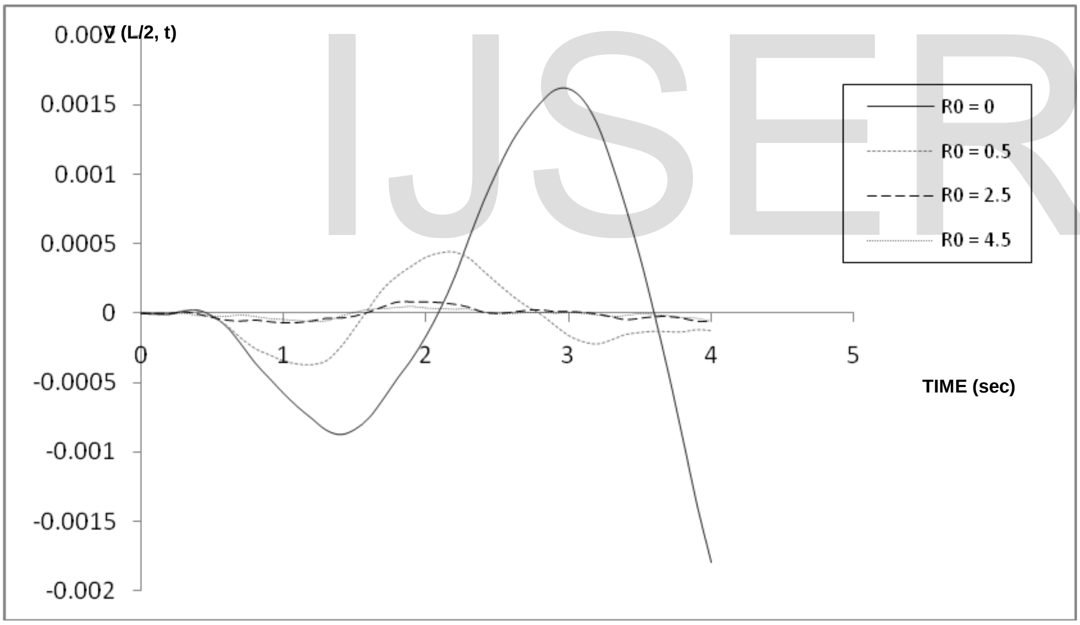

Fig 4: Deflection profile of a simply supported Rayleigh beam under the actions of concentrated masses travelling at constant velocity for various values of Rotatory inertia and for fixed value of foundation modulus K_0 = 400000 and shear modulus G_0 = 90000.

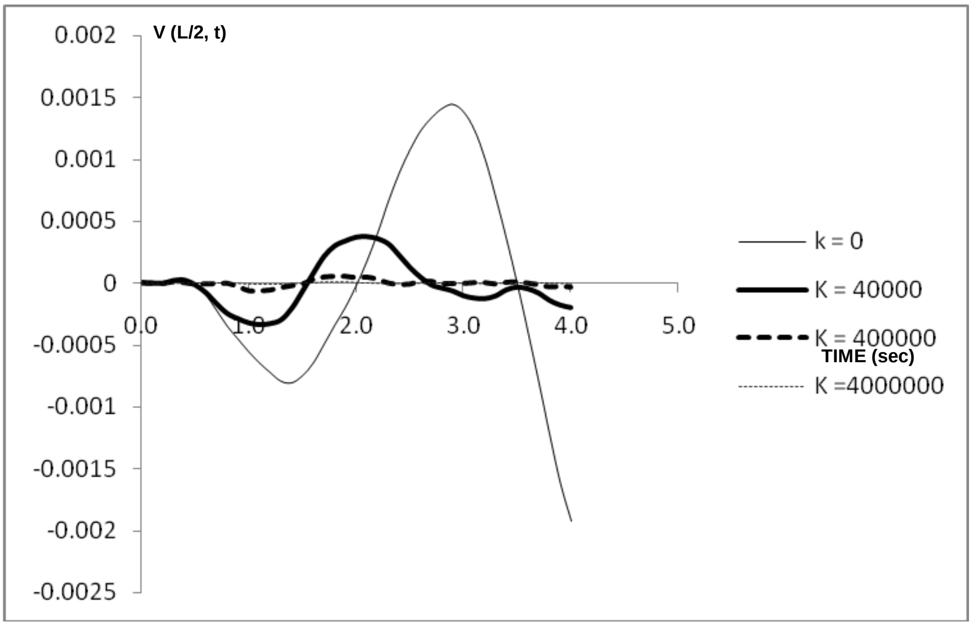

Fig 5: Response amplitude of a simply supported Rayleigh beam under the action of concentrated mass travelling at constant velocity for various values of Foundation Modulus K_0 and for fixed values of Shear Modulus G_0 = 40,000 and Rotatory inertia R_o = 2.5.

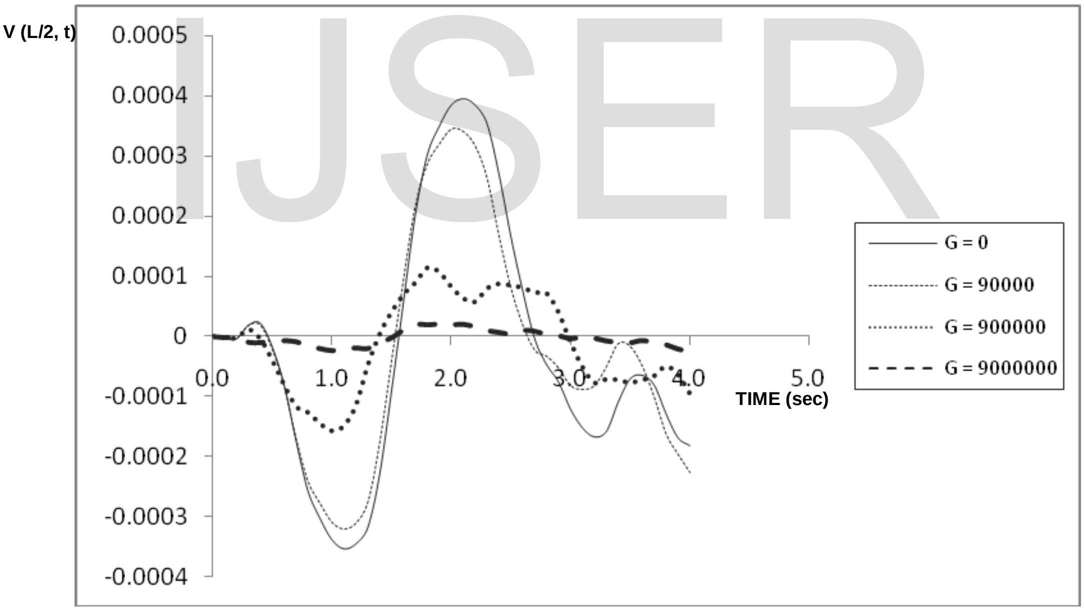

Fig 6: Response amplitude of a simply supported Rayleigh beam under the action of concentrated mass travelling at constant velocity for various values of Shear Modulus G_0 and for fixed values of Foundation Modulus K_0 = 40,000 and Rotatory inertia R_o =2.5.

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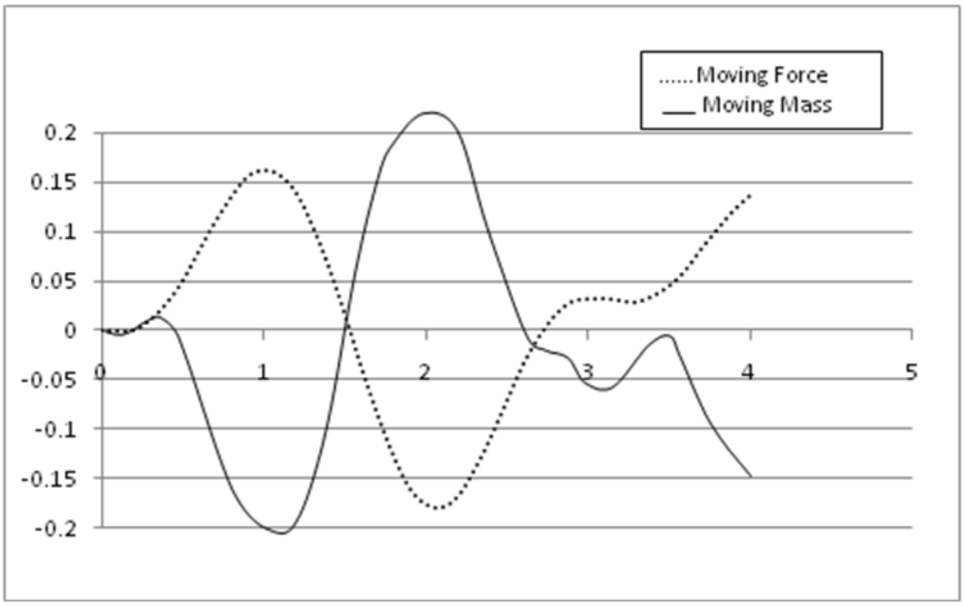

Fig 7: Comparism of the displacement response of moving force and moving mass of uniform simply supported Rayleigh beam for fixed values of Rotatory inertia R_O = 2.5, K_0 = 400000 and G_0 =90000.

From the graphs above, figures (1) and (4) displays the effect of Rotatory inertia R_O on the transverse deflection of the simply supported uniform Rayleigh beam in both cases of moving force and moving mass problems respectively. The graphs show that the response amplitude increases as the value of the Rotatory inertia decreases.

Figures (2) and (5) displays the effect of foundation modulus (K_0) on the transverse deflection of simply supported uniform Rayleigh beam in both cases of moving force and moving mass respectively. The graph shows that an increase in the Rotatory inertia resulted to decrease in the amplitude of vibration

Figures (3) and (6) shows the influence of shear modulus (G_0) on the deflection profile of simply supported uniform Rayleigh beam in both cases of moving force and moving mass problems respectively. The graphs show that higher values of shear modulus decrease the vibration of the beam.

Figure (7) compares the displacement response of moving force and moving mass and from the graph, the response amplitude of moving mass problem is higher than that of the moving force problem of simply supported uniform Rayleigh beam for a fixed values of Rotatory inertia shear modulus (G_0) and foundation modulus (K_0)

Table 1: Results for various values of rotatory inertia $R_{O,}$ with fixed values of shear modulus G_O = 900,000 and foundation modulus K_O = 400,000 for both cases of moving force and moving mass

MOVING FORCE					MOVING MASS			
T(sec)	R0 = 0	R0 = 0.5	R0 = 1.5	R0=2.5	R0 = 0	R0 = 0.5	R0 = 2.5	R0 = 4.5
0	0	0	0	0	0	0	0	0
0.1	6.97E-04	6.94E-04	6.87E-04	6.80E-04	-3.54E-06	-3.47E-06	-3.21E-06	-2.96E-06
0.2	1.53E-03	1.49E-03	1.43E-03	1.37E-03	-4.89E-06	-4.24E-06	-2.23E-06	-9.86E-07
0.3	1.87E-03	1.77E-03	1.57E-03	1.38E-03	1.88E-05	1.70E-05	9.54E-06	3.08E-06
0.4	2.38E-03	2.15E-03	1.73E-03	1.38E-03	2.48E-05	1.57E-05	-6.91E-06	-1.38E-05
0.5	2.45E-03	2.03E-03	1.36E-03	8.46E-04	-1.68E-05	-2.59E-05	-3.16E-05	-1.97E-05
0.6	2.42E-03	1.80E-03	8.77E-04	2.88E-04	-9.40E-05	-9.06E-05	-5.20E-05	-1.91E-05
0.7	2.32E-03	1.49E-03	4.12E-04	-1.18E-04	-2.18E-04	-1.82E-04	-5.95E-05	-1.02E-05
0.8	1.66E-03	6.49E-04	-4.19E-04	-7.44E-04	-3.55E-04	-2.57E-04	-5.14E-05	-2.09E-05
0.9	1.16E-03	4.42E-05	-8.24E-04	-8.36E-04	-4.66E-04	-3.03E-04	-6.39E-05	-3.64E-05
1.0	2.96E-04	-8.07E-04	-1.29E-03	-9.65E-04	-5.75E-04	-3.47E-04	-7.23E-05	-4.28E-05
1.1	-3.61E-04	-1.32E-03	-1.28E-03	-6.95E-04	-6.74E-04	-3.70E-04	-7.09E-05	-5.12E-05
1.2	-9.20E-04	-1.60E-03	-9.77E-04	-2.92E-04	-7.58E-04	-3.74E-04	-5.83E-05	-5.41E-05
1.3	-1.73E-03	-2.00E-03	-8.68E-04	-2.60E-04	-8.40E-04	-3.46E-04	-3.76E-05	-5.22E-05
1.4	-2.11E-03	-1.89E-03	-3.92E-04	-4.13E-06	-8.72E-04	-2.46E-04	-3.68E-05	-1.87E-05
1.5	-2.58E-03	-1.81E-03	-1.95E-04	-8.87E-05	-8.35E-04	-1.08E-04	-2.16E-05	1.07E-05
1.6	-2.51E-03	-1.23E-03	2.51E-04	1.15E-04	-7.50E-04	3.94E-05	9.53E-06	2.96E-05
1.7	-2.23E-03	-5.26E-04	5.79E-04	3.16E-04	-6.14E-04	1.79E-04	4.94E-05	3.55E-05
1.8	-2.04E-03	-6.19E-05	4.98E-04	2.49E-04	-4.65E-04	2.72E-04	8.40E-05	4.40E-05
1.9	-1.41E-03	6.51E-04	5.99E-04	4.70E-04	-3.26E-04	3.43E-04	8.45E-05	5.13E-05
2.0	-9.36E-04	9.78E-04	3.52E-04	3.72E-04	-1.60E-04	4.07E-04	8.46E-05	3.96E-05
2.1	-3.64E-05	1.51E-03	4.47E-04	5.57E-04	3.47E-05	4.38E-04	7.96E-05	3.48E-05
2.2	7.76E-04	1.76E-03	4.54E-04	5.33E-04	2.65E-04	4.43E-04	6.69E-05	3.05E-05
2.3	1.32E-03	1.60E-03	2.63E-04	1.86E-04	5.39E-04	3.99E-04	4.48E-05	3.19E-05
2.4	2.00E-03	1.49E-03	3.24E-04	2.39E-05	8.02E-04	3.06E-04	7.10E-06	1.26E-05
2.5	2.19E-03	9.23E-04	4.86E-05	-4.47E-04	1.03E-03	2.15E-04	-1.57E-06	-1.01E-06
2.6	2.54E-03	6.15E-04	1.03E-04	-4.63E-04	1.24E-03	1.29E-04	7.04E-06	8.76E-07
2.7	2.51E-03	1.06E-04	-4.72E-05	-5.00E-04	1.39E-03	5.37E-05	2.06E-05	7.71E-06
2.8	2.17E-03	-4.70E-04	-3.19E-04	-4.89E-04	1.52E-03	-6.27E-06	2.58E-05	1.04E-05
2.9	1.87E-03	-7.42E-04	-3.65E-04	-1.58E-04	1.61E-03	-8.71E-05	1.13E-05	2.43E-06
3.0	1.06E-03	-1.23E-03	-7.12E-04	-1.61E-04	1.62E-03	-1.63E-04	1.19E-05	8.46E-06
3.1	5.03E-04	-1.22E-03	-6.24E-04	1.22E-04	1.54E-03	-2.08E-04	7.49E-06	1.36E-06
3.2	-3.05E-04	-1.27E-03	-6.37E-04	8.04E-05	1.37E-03	-2.24E-04	-6.12E-06	-9.43E-06
3.3	-1.05E-03	-1.14E-03	-4.94E-04	-2.36E-05	1.09E-03	-1.94E-04	-2.69E-05	-2.18E-05
3.4	-1.52E-03	-7.19E-04	-7.80E-05	1.40E-05	7.68E-04	-1.55E-04	-4.91E-05	-1.30E-05
3.5	-2.21E-03	-5.87E-04	1.75E-05	-2.67E-04	4.08E-04	-1.41E-04	-3.83E-05	-1.89E-07
3.6	-2.38E-03	-1.03E-04	4.22E-04	-9.75E-05	1.95E-06	-1.32E-04	-2.69E-05	-5.96E-06
3.7	-2.59E-03	1.21E-04	5.04E-04	-2.25E-05	-4.30E-04	-1.35E-04	-2.55E-05	-1.49E-05
3.8	-2.41E-03	4.55E-04	6.33E-04	3.30E-04	-8.98E-04	-1.36E-04	-3.85E-05	-2.85E-05
3.9	-1.89E-03	8.30E-04	7.49E-04	8.15E-04	-1.38E-03	-1.20E-04	-6.11E-05	-3.44E-05

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4.0	-1.57E-03	7.46E-04	3.84E-04	8.17E-04	-1.79E-03	-1.28E-04	-6.31E-05	-5.52E-05

Table 2: Results for various values of foundation modulus K_O, with fixed values of shear modulus G_O =900,000 and rotatory inertia R_O for both cases of moving force and moving mass

MOVING FORCE					**MOVING MASS**			
T(sec)	K = 0	K = 40000	K = 400000	K = 4000000	K = 0	K = 40000	K = 400000	K = 4000000
0	0	0	0	0	0	0	0	0
0.1	-6.80E-04	-6.77E-04	-6.50E-04	-4.29E-04	-3.53E-06	-3.51E-06	-3.36E-06	-2.18E-06
0.2	4.65E-05	5.06E-05	7.53E-05	-1.72E-04	-4.76E-06	-4.65E-06	-3.84E-06	-1.79E-06
0.3	1.06E-02	1.02E-02	6.88E-03	-3.12E-04	1.86E-05	1.76E-05	1.03E-05	-2.68E-06
0.4	2.91E-02	2.69E-02	1.28E-02	8.01E-04	2.31E-05	2.00E-05	2.28E-06	6.15E-08
0.5	5.56E-02	4.95E-02	1.73E-02	1.33E-03	-1.92E-05	-1.94E-05	-9.66E-06	2.24E-06
0.6	9.39E-02	7.97E-02	1.92E-02	2.29E-03	-9.58E-05	-8.21E-05	-6.97E-06	2.66E-06
0.7	0.1368906	0.108647	1.63E-02	2.79E-03	-2.17E-04	-1.73E-04	-5.61E-06	4.70E-06
0.8	0.1833918	0.1342565	1.57E-02	2.66E-03	-3.47E-04	-2.51E-04	-3.19E-06	1.70E-07
0.9	0.233009	0.154658	1.54E-02	2.31E-03	-4.50E-04	-2.92E-04	-2.94E-05	-1.34E-06
1.0	0.276621	0.1612865	0.0148814	1.48E-03	-5.53E-04	-3.25E-04	-6.14E-05	-7.05E-06
1.1	0.3137915	0.1561683	1.47E-02	4.64E-04	-6.44E-04	-3.39E-04	-6.32E-05	-1.01E-05
1.2	0.3417625	0.1387919	9.04E-03	1.54E-04	-7.22E-04	-3.30E-04	-5.57E-05	-1.11E-05
1.3	0.3517773	0.1058931	3.38E-03	-6.65E-04	-7.92E-04	-3.00E-04	-3.55E-05	-1.01E-05
1.4	0.3450527	6.49E-02	-9.42E-04	-1.75E-05	-8.05E-04	-2.07E-04	-2.13E-05	-5.86E-06
1.5	0.31788	1.71E-02	-3.67E-03	-2.23E-04	-7.49E-04	-6.82E-05	-7.65E-06	7.73E-07
1.6	0.2649225	-3.56E-02	-1.29E-03	4.26E-04	-6.46E-04	7.54E-05	2.78E-05	4.47E-06
1.7	0.1915606	-8.35E-02	-1.34E-03	3.78E-04	-4.96E-04	2.11E-04	4.73E-05	1.15E-05
1.8	9.70E-02	-0.1259384	-4.13E-03	-3.38E-05	-3.41E-04	2.96E-04	5.35E-05	9.79E-06
1.9	-0.0164091	-0.158944	-9.14E-03	-4.51E-04	-1.95E-04	3.36E-04	5.18E-05	1.12E-05
2.0	-0.1374477	-0.1752707	-1.73E-02	-1.61E-03	-2.07E-05	3.66E-04	4.29E-05	5.85E-06
2.1	-0.2627826	-0.1785965	-0.0203756	-2.19E-03	1.78E-04	3.70E-04	4.66E-05	2.44E-06
2.2	-0.3839211	-0.1671378	-2.02E-02	-2.73E-03	4.06E-04	3.53E-04	2.84E-05	-1.38E-06
2.3	-0.4882285	-0.1407805	-1.77E-02	-2.83E-03	6.62E-04	3.12E-04	-9.15E-07	-3.51E-06
2.4	-0.5732464	-0.1086789	-1.30E-02	-2.12E-03	8.88E-04	2.20E-04	-1.06E-05	-3.86E-06
2.5	-0.6304917	-7.17E-02	-1.26E-02	-1.57E-03	1.08E-03	1.22E-04	-8.55E-06	-1.04E-06
2.6	-0.6525543	-3.48E-02	-1.18E-02	-5.39E-04	1.23E-03	4.32E-05	1.14E-05	-1.13E-06
2.7	-0.641828	-6.50E-03	-9.85E-03	1.45E-04	1.33E-03	-1.88E-05	1.27E-05	3.66E-06
2.8	-0.5934159	0.0156402	-7.27E-03	1.99E-04	1.41E-03	-4.46E-05	-4.65E-06	9.58E-07
2.9	-0.5074162	2.87E-02	-6.26E-04	5.75E-04	1.44E-03	-6.77E-05	-4.67E-06	2.87E-06
3.0	-0.3912755	3.15E-02	3.07E-03	-3.07E-04	1.38E-03	-1.06E-04	-3.88E-06	-6.30E-07
3.1	-0.2448461	3.21E-02	4.40E-03	-3.78E-05	1.23E-03	-1.25E-04	2.84E-06	-1.64E-06
3.2	-7.64E-02	0.0298781	4.89E-03	-5.68E-04	9.96E-04	-1.29E-04	1.90E-06	-2.33E-06
3.3	0.1020834	2.83E-02	3.55E-03	1.24E-05	6.82E-04	-1.07E-04	-7.71E-06	-2.12E-06
3.4	0.2854898	0.0344556	7.59E-03	6.24E-04	3.46E-04	-5.79E-05	4.96E-06	-4.26E-07
3.5	0.4593943	4.44E-02	1.36E-02	1.39E-03	-1.53E-05	-3.72E-05	8.37E-06	2.72E-06
3.6	0.6109798	5.99E-02	0.018969	2.35E-03	-4.07E-04	-4.98E-05	-1.17E-06	2.34E-06
3.7	0.7348408	8.18E-02	2.31E-02	2.71E-03	-8.09E-04	-8.45E-05	-1.41E-05	4.87E-06
3.8	0.8177841	0.1019367	2.00E-02	2.68E-03	-1.23E-03	-1.43E-04	-3.10E-05	1.14E-07

3.9	0.8543453	0.1209015	1.53E-02	2.40E-03	-1.62E-03	-1.81E-04	-2.63E-05	-1.51E-06
4.0	0.84542	0.135534	0.0111612	1.24E-03	-1.92E-03	-2.01E-04	-3.32E-05	-6.88E-06

Table 3: Results for various values of shear modulus $G_{O,}$ with fixed values of foundation modulus K_O = 400,000 and rotatory inertia R_O = 2.5 for both cases of moving force and moving mass

MOVING FORCE					**MOVING MASS**			
T(sec)	G = 0	G = 90000	G = 900000	G = 9000000	G = 0	G = 90000	G = 900000	G = 9000000
0	0	0	0	0	0	0	0	0
0.1	-6.84E-04	-6.77E-04	-6.16E-04	-1.66E-04	-3.52E-06	-3.50E-06	-3.24E-06	-1.46E-06
0.2	-4.21E-05	5.06E-05	7.70E-04	2.35E-03	-4.79E-06	-4.49E-06	-2.33E-06	-1.70E-06
0.3	1.01E-02	1.02E-02	0.0103067	6.14E-03	1.79E-05	1.73E-05	1.11E-05	-9.38E-06
0.4	2.72E-02	2.69E-02	2.39E-02	8.95E-03	2.16E-05	1.80E-05	-5.72E-06	-9.65E-06
0.5	5.03E-02	4.95E-02	4.29E-02	9.59E-03	-1.74E-05	-2.17E-05	-4.12E-05	-7.80E-06
0.6	8.16E-02	7.97E-02	6.33E-02	8.92E-03	-8.09E-05	-8.34E-05	-8.24E-05	-6.63E-06
0.7	0.1127152	0.108647	7.83E-02	9.80E-03	-1.74E-04	-1.70E-04	-1.17E-04	-8.47E-06
0.8	0.1406675	0.1342565	8.83E-02	1.20E-02	-2.58E-04	-2.42E-04	-1.27E-04	-1.60E-05
0.9	0.1645566	0.154658	8.66E-02	1.22E-02	-3.03E-04	-2.78E-04	-1.45E-04	-1.96E-05
1.0	0.1753645	0.1612865	7.53E-02	9.77E-03	-3.39E-04	-3.09E-04	-1.57E-04	-2.29E-05
1.1	0.1737616	0.1561683	5.69E-02	6.08E-03	-3.54E-04	-3.21E-04	-1.48E-04	-1.97E-05
1.2	0.1601246	0.1387919	3.20E-02	3.54E-03	-3.46E-04	-3.11E-04	-1.14E-04	-1.85E-05
1.3	0.1295818	0.1058931	9.28E-03	3.56E-03	-3.19E-04	-2.77E-04	-4.74E-05	-1.94E-05
1.4	0.0882535	6.49E-02	-1.12E-02	3.49E-03	-2.31E-04	-1.78E-04	3.72E-06	-1.01E-05
1.5	3.86E-02	1.71E-02	-2.54E-02	9.10E-04	-9.19E-05	-4.21E-05	4.06E-05	-1.75E-06
1.6	-0.0195209	-3.56E-02	-3.12E-02	-2.63E-03	5.69E-05	9.42E-05	6.91E-05	1.12E-05
1.7	-7.61E-02	-8.35E-02	-3.39E-02	-5.08E-03	2.03E-04	2.17E-04	8.91E-05	2.01E-05
1.8	-0.1281869	-0.1259384	-3.15E-02	-5.66E-03	3.02E-04	2.86E-04	1.14E-04	1.95E-05
1.9	-0.1733085	-0.158944	-3.03E-02	-5.60E-03	3.49E-04	3.18E-04	1.06E-04	2.06E-05
2.0	-0.2017483	-0.1752707	-3.23E-02	-7.71E-03	3.84E-04	3.43E-04	8.18E-05	1.95E-05
2.1	-0.2143949	-0.1785965	-3.64E-02	-1.13E-02	3.95E-04	3.40E-04	6.29E-05	2.04E-05
2.2	-0.2108349	-0.1671378	-4.65E-02	-1.30E-02	3.84E-04	3.16E-04	5.69E-05	1.62E-05
2.3	-0.187454	-0.1407805	-5.62E-02	-1.18E-02	3.53E-04	2.62E-04	7.90E-05	1.02E-05
2.4	-0.1524134	-0.1086789	-6.55E-02	-9.22E-03	2.70E-04	1.66E-04	8.70E-05	7.08E-06
2.5	-0.1092716	-7.17E-02	-7.13E-02	-7.39E-03	1.65E-04	8.00E-05	8.41E-05	4.66E-06
2.6	-6.03E-02	-3.48E-02	-6.82E-02	-7.56E-03	7.43E-05	1.74E-05	8.04E-05	1.06E-05
2.7	-1.64E-02	-6.50E-03	-5.96E-02	-7.17E-03	-5.07E-06	-2.38E-05	7.23E-05	8.89E-06
2.8	2.08E-02	0.0156402	-4.20E-02	-4.15E-03	-4.78E-05	-3.39E-05	7.12E-05	3.97E-06
2.9	0.0503431	2.87E-02	-1.92E-02	-5.17E-04	-7.67E-05	-5.35E-05	3.97E-05	1.81E-06
3.0	6.61E-02	3.15E-02	4.51E-03	1.64E-03	-1.22E-04	-8.26E-05	-9.22E-06	-3.05E-06
3.1	7.37E-02	3.21E-02	2.91E-02	1.85E-03	-1.53E-04	-8.89E-05	-5.05E-05	-6.81E-07
3.2	0.0751733	0.0298781	4.69E-02	1.67E-03	-1.68E-04	-8.25E-05	-7.71E-05	-2.38E-06
3.3	6.96E-02	2.83E-02	5.92E-02	4.05E-03	-1.59E-04	-5.10E-05	-7.14E-05	-6.83E-06
3.4	6.56E-02	0.0344556	6.44E-02	8.25E-03	-1.09E-04	-1.27E-05	-7.23E-05	-9.55E-06
3.5	6.35E-02	4.44E-02	6.14E-02	1.08E-02	-7.09E-05	-1.76E-05	-7.70E-05	-1.02E-05
3.6	6.26E-02	5.99E-02	5.64E-02	1.09E-02	-6.51E-05	-5.00E-05	-7.26E-05	-6.33E-06
3.7	0.0688315	8.18E-02	4.78E-02	9.92E-03	-8.15E-05	-1.02E-04	-6.67E-05	-9.39E-06

3.8	7.81E-02	0.1019367	4.11E-02	9.71E-03	-1.29E-04	-1.63E-04	-5.03E-05	-1.30E-05
3.9	8.82E-02	0.1209015	3.81E-02	1.10E-02	-1.69E-04	-1.97E-04	-6.24E-05	-2.00E-05
4.0	0.1017883	0.135534	3.65E-02	1.10E-02	-1.82E-04	-2.27E-04	-9.39E-05	-2.48E-05

Table 4: Result comparism of the displacement response of moving force and moving mass of uniform simply supported Rayleigh beam for fixed values of Rotatory inertia R_0 = 2.5, K_0 = 400000 and G_0 =90000.

T(sec)	MOVING FORCE	MOVING MASS
0	0	0
0.1	-5.35E-06	-6.50E-04
0.2	5.71E-06	7.53E-05
0.3	5.00E-05	6.88E-03
0.4	1.41E-06	1.28E-02
0.5	-9.83E-06	1.73E-02
0.6	-6.77E-06	1.92E-02
0.7	-5.00E-06	1.63E-02
0.8	-3.10E-06	1.57E-02
0.9	-3.05E-05	1.54E-02
1.0	-6.06E-05	0.014881
1.1	-6.15E-05	1.47E-02
1.2	-5.44E-05	9.04E-03
1.3	-3.50E-05	3.38E-03
1.4	-2.27E-05	-9.42E-04
1.5	-6.95E-06	-3.67E-03
1.6	2.89E-05	-1.29E-03
1.7	4.59E-05	-1.34E-03
1.8	5.21E-05	-4.13E-03
1.9	5.01E-05	-9.14E-03
2.0	4.42E-05	-1.73E-02
2.1	4.79E-05	-0.02038
2.2	2.54E-05	-2.02E-02
2.3	-2.16E-06	-1.77E-02
2.4	-1.01E-05	-1.30E-02
2.5	-5.74E-06	-1.26E-02
2.6	1.35E-05	-1.18E-02
2.7	8.94E-06	-9.85E-03
2.8	-5.58E-06	-7.27E-03
2.9	-2.94E-06	-6.26E-04
3.0	-2.55E-06	3.07E-03
3.1	3.50E-06	4.40E-03
3.2	-1.90E-06	4.89E-03
3.3	-7.09E-06	3.55E-03
3.4	8.12E-06	7.59E-03
3.5	6.81E-06	1.36E-02
3.6	-2.96E-06	0.018969
3.7	-1.69E-05	2.31E-02

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3.8	-2.79E-05	2.00E-02
3.9	-2.09E-05	1.53E-02
4.0	2.61E-05	0.011161

7. CONCLUSION

The problem of vibrations of uniform Rayleigh beam resting on elastic Pasternak foundation and transverse by concentrated masses travelling at constant velocity has been investigated. Illustrative example involving simply supported is presented. The solutions hitherto obtained are analyzed and resonance conditions for the various problems are established. Results show that:

Resonance is reached earlier in a system traversed by moving mass then in that under the action of a moving force.

1. As the shear modulus (G), Rotatory inertia (R_0) and foundation modulus (K) increases, the amplitude of uniform Rayleigh beam under the action of moving loads moving at constant velocity decreases.

2. When the values of the shear modulus (G) and Rotatory inertia (R_0) are fixed, the displacement of uniform Rayleigh beam resting on elastic Pasternak foundation and traversed by masses travelling with constant velocity.

3. For fixed value of axial force, shear modulus and foundation modulus, the response amplitude for the moving mass problem is greater than that of the moving force problem for the illustrated end condition considered.

4. It has been established that, the moving force solution is not an upper bound for accurate solution of the moving mass in uniform Rayleigh beams under accelerating loads. Hence, the non- reliability of moving force solution as a safe approximation to the moving mass problem is confirmed.

5. In the illustrated examples, for the same natural frequency, the critical velocity for moving mass problem is smaller than that of the moving force problem. Hence, resonance is reach earlier in the moving mass problem.

References

[1] Aiyesimi Y.M (1990): Dynamic response of elastic structures continuously supported visco-effect foundation and traversed by moving loads. M.sc thesis university of Ilorin.

[2] Fryba, L (1972): vibrations of solids and structures under moving loads Groningen: Noordhoff,

[3] Gbadeyan J.A and Oni, S.T (1992): dynamic response to moving concentrated masses of elastic plates on a non-Winkler elastic foundation. Journal of sound and vibration vol 54, pp 343-358.

[4]Gbadeyan, J.A and Oni S.T (1995): Dynamic behavior of beams and rectangular plates under moving loads. Journal of sound and vibration 182(5), pp 677-695.

[5] Kenny .J (1954): Steady state vibrations of a beam on an elastic foundation for a moving load. Journal of Applied Mechanics (ASME) Vol. 76.

[6] Milormir M; Stanisic, M.M; Hardin J.C (1969): On the response of beam to an arbitrary number of concentrated moving masses. Journal of the Franklin institute, vol 287 No 2.

[7] Mindhin R.D (1951): influence of rotary inertia and shear modulus on flexural motions of isotropic elastic plates. Journal of Applied Mechanics pp 31- 38.

[8] Odman, S. T (1968): Differential Equation for calculation of vibrations produced in load- bearing structures by moving loads, preliminary publication. International Association for bridges and structural Engineering, 3rd congress, Liege, pp 669- 680

[9] Oni S.T and T.O Awodola (2003): vibrations under a moving of a non-uniform Rayleigh beam on variable elastic foundation Vol 7, pp 191-206, Journal of the Nigerian association of Mathematical Physics.

[10] Pasternak, P.I. (1954): On a new method of an elastic foundation by means of two foundation constants (in Russian). Gosudarstennue izdatelstve literaturi Postoitelstrui Arkhitekture, Moscow.

[11] Stanisic, M.M; Hardin, J.C **(**1968**):** On the response of plate to a moving multi-masses moving system. Acta mechanical 5, 37-53.

[12] Timoshenko, S (1952): On the transverse vibrations of bars of uniform cross section. Philosophy mag. Ser. 6, vol.43, pp.125 – 131.

[13] Willis, R. (1951): Preliminary essay to the appendix B, Experiments for determining the effects produced by causing weights to travel over bars with different velocities. Barlow, P., Treatie on the strength of timber, cast iron and malleable iron. London

[14] Winkler, E (1982): Die Lahre vonder Elastizitiit and festigkeit Prague.